Properties of Ellipse.

1) Since the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ contains only even powers of 'x' & 'y' therefore it is symmetrical w.r.t. x-axis and y-axis.

2) The ellipse cuts x-axis at two points i.e., at $x = \pm a$. and y-axis at two points $y = \pm b$

3) An ellipse has two focus points (foci) i.e., $(-ae, 0)$ and $(ae, 0.)$ and the equation of directrix respective are $x = \frac{-a}{e}$ & $x = \frac{a}{e}$

4) The focal chord is given by $(a + ex)$

5) In an ellipse chord AA_1 is called major-axis and chord BB_1 is called minor axis. And LSL_1 is called the latus rectum with the eqn $x = -ae$.

Length of latus rectum $= \frac{2b^2}{a}$

Ques Find the foci, directrices & eccentricity of the ellipse $3x^2 + 4y^2 = 12$.

Soln $3x^2 + 4y^2 = 12 \Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$

$a^2 = 4$, $b^2 = 3$

$\Rightarrow a = 2.$

We know

$b^2 = a^2(1-e^2)$

$3 = 4(1-e^2) \Rightarrow 1-e^2 = \frac{3}{4}$

$\Rightarrow e^2 = 1-\frac{3}{4} = \frac{1}{4} \Rightarrow e = \frac{1}{2}$

Focus $\Rightarrow (-ae, 0), (ae, 0)$

$ae = 2 \cdot \frac{1}{2} = 1 \Rightarrow$ Focus $= (-1, 0); (1; 0)$

Directrix $= \pm \frac{a}{e} = \pm \frac{2}{1/2} = \pm 4$

$\Rightarrow x = \pm 4.$

Ques Find the eccentricity & centre of the ellipse $2x^2 + 3y^2 - 4x + 5y + 4 = 0$

Soln Given, $2x^2 + 3y^2 - 4x + 5y + 4 = 0.$

$2x^2 - 4x + 3y^2 + 5y + 4 = 0$

$\Rightarrow 2(x^2 - 2.x + 1) + 3(y^2 + \frac{5}{3}y + \frac{25}{36}) = 2 + \frac{25}{12} - 4$

$\Rightarrow 2(x-1)^2 + 3(y+5/6)^2 = \frac{25 + 24 - \cancel{60}\,48}{12} = \frac{1}{12}$

$\Rightarrow \frac{(x-1)^2}{1/24} + \frac{(y+5/6)^2}{1/36} = 1$ — ①

Put $x-1 = X$ and $y + 5/6 = Y$

$\Rightarrow \frac{X^2}{1/24} + \frac{Y^2}{1/36} = 1$ — ②

For eqn ① origin is $(0,0)$ but for eqn ② then changes to $(1, -5/6)$ which is the required centre.

$a^2 = 1/24$, $b^2 = 1/36 \Rightarrow b^2 = a^2(1-e^2)$

$\Rightarrow 1 - e^2 = \frac{\cancel{24}\,2}{\cancel{36}\,3} = \frac{2}{3} \Rightarrow e^2 = 1 - \frac{2}{3}$

$\Rightarrow e^2 = \frac{1}{3} \Rightarrow e = \frac{1}{\sqrt{3}}.$

Questions

1) Find the co-ordinates of the second focus and equation of second directrix for the ellipse whose one focus is $S(1,2)$ and the corresponding directrix is the line $x-y=5$ and $e=1/2$.

2) Find Eqn of ellipse whose focus is $(2,4)$, eccentricity $1/\sqrt{13}$ and directrix $x-y+13=0$

3) Find the eqn to the ellipse whose centre is the origin, whose axes are the axes of co-ordinates & which pass through.
(i) Points $(2,1)$ & $(4,-3)$
(ii) the points $(1,6)$ & $(2,-5)$

4) The latus rectum of an ellipse is 5 and its eccentricity is $1/3$. Find its eqn referred to its axes ~~of~~ as the axes of co-ordinates.

5) Find the eccentricity, latus rectum & the focus of the ellipse in each case
(i) $4x^2+y^2-8x+2y+1=0$ (ii) $7x^2+6y^2-42x-24y+86=0$

6) Find the centre, length of axes and the eccentricity of the ellipse

$$2x^2+3y^2-4x-12y+13=0$$